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BRIEF COMMUNICATION

ON FRICTIONAL PRESSURE GRADIENT IN ANNULAR FLOW

M. FØRDE† and H. NØRSTRUD
Division of Aero- and Gas Dynamics, The Norwegian Institute of Technology, N-7034—Trondheim-NTH, Norway

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Vertical annular two-phase flow has been studied experimentally and from a more fundamental fluid mechanics point of view. The purpose has often been in connection with pressure drop and heat transfer. The present note is based on pressure drop in one-phase pipeflow and the idea in the present model is to model the effective roughness height on the liquid film, the results are compared with air—water data and the calculation fits the data satisfactorily.

A considerable simplification in the calculation of frictional pressure drop can be made by making the following assumptions:

—that the wall shear stress is the same as when only the liquid is flowing in the pipe with the same mean velocity

—that all of the liquid flowing in the pipe flows in the film

In addition to these two assumptions, we know that the pressure gradient is equal for both phases of the fluid. The frictional pressure gradient for the liquid-phase can be calculated by:

$$\left(\frac{\mathrm{d}p_f}{\mathrm{d}z}\right)_L = \lambda_L \frac{\rho_L}{2\cdot d}\bar{u}_L{}^2 \qquad [1]$$

where ρ_L = liquid density, d = tube diameter, $\bar{u}_L$ = mean liquid film velocity and where the liquid phase friction factor, λ_L, is dependent on the Reynolds number and the roughness of the pipe wall.

The liquid film Reynolds number is

$$\mathrm{Re}_L = \frac{\rho_L\cdot d\cdot \bar{u}_L}{\mu_L}$$

where μ_L = liquid viscosity, and the mean velocity of the liquid takes the form

$$\bar{u}_L = \frac{\dot{m}_L}{\rho_L\cdot(1-\epsilon)}$$

where $\dot{m}_L$ = liquid mass-flux. Here the void-fraction is defined as $\epsilon = A_G/A$ where A = tube cross-sectional area, A_G = cross-sectional area occupied by the gas-phase.

The frictional pressure gradient of the gas-phase can be calculated from the relation

$$\left(\frac{\mathrm{d}p_f}{\mathrm{d}z}\right)_G = \lambda_G\cdot\frac{\rho_G}{2(d-2\delta)}\bar{u}_G{}^2 \qquad [2]$$

†This author wishes to express his sincere thanks to Dr. G. F. Hewitt and colleagues for the stay at Harwell during the summer 1982, when the idea to the present model was created.

where ρ_G = gas-density, δ = liquid film thickness and where the mean gas velocity

$$u_G = \frac{\dot{m}_G}{\rho_G \cdot \epsilon}$$

and the gas-phase Reynolds number

$$\mathrm{Re}_G = \frac{\rho_G(d - 2\delta)\bar{u}_G}{\mu_G}$$

where μ_G = gas-viscosity.

The interfacial friction factor, λ_G, is dependent on the gas-phase Reynolds number and the effective roughness height on the liquid film. The remaining problem is to find the effective roughness height.

We first assume a relationship between the gas frictional force (per unit mass) and the liquid surface tension of the form

$$\frac{\mu_G u_G}{\rho_G l_G^2} = \alpha^2 \frac{\sigma}{\rho_L l_L^2} \quad [3]$$

where the proportionality factor α could be obtained from experimental observations.

With the following definition for the two length scales, i.e.

$$l_L = k/2$$

$$l_G = 2\delta$$

[3] yields

$$\left(\frac{k}{\delta}\right)_{\mathrm{eff}} = 4\alpha\left(\frac{\rho_G}{\rho_L}\frac{\sigma}{\mu_G u_G}\right)^{1/2}. \quad [4]$$

Here the length scale l_L for the liquid phase is equal to the amplitude for the interfacial wave (or half the roughness length k). Furthermore the length scale l_G is the diameter of the gas core (hence the factor 2 in the above expression) and this dimension is of the same order of magnitude as the thickness δ.

If we now proceed one step further and postulate a direct correspondance between the two forces per unit pipe length acting on the interface, i.e.

$$\rho_G A_G \frac{\mu_G u_G}{\rho_G l_G^2} = \rho_L(A - A_G)\frac{\sigma}{\rho_L l_L^2}$$

the factor α can, by comparing the above equation with [3], be uniquely defined as

$$\alpha = \sqrt{\frac{1-\epsilon}{\epsilon}\frac{\rho_L}{\rho_G}} \quad [5]$$

where the mass per unit length of the gas and liquid phase are respectively $\rho_G A_G$ and $\rho_L(A - A_G)$. The void fraction ϵ yields, with [4], the properties

$$\epsilon \to 1 \text{ (gas only): } (k/\delta)_{\mathrm{eff}} \to 0$$

$$\epsilon \to 0 \text{ (liquid only): } (k/\delta)_{\mathrm{eff}} \to \infty$$

and this implies an increase of the gas frictional force (or of the effective roughness height k) with an increase of the liquid fraction (decreasing ϵ).

With the above stated assumption the factor α can be eliminated, i.e. combining [4] and [5] gives the result

$$\left(\frac{k}{\delta}\right)_{\text{eff}} = 4\sqrt{\frac{(1-\epsilon)}{\epsilon}}\left(\frac{\sigma}{\mu_G u_G}\right)^{1/2} \qquad [6]$$

As it can be seen, the effective roughness height decreases with reduced value for the surface tension σ. This is in accordance with the experience of reducing ocean waves by adding oil to the surface.

It should be noted that [6] bears a resemblance to the expression given by Wallis (on p. 320) in which the r.h.s. of [6] only contains the factor 4. Hence, the present formulation can be regarded as generalization of Wallis formulation.

When we know the Reynolds number and the effective roughness height, we can find the interfacial friction factor by an explicite formula. Here we calculate the friction factor by the explicit formula developed by Haaland (1981), i.e.

$$\frac{1}{\sqrt{\lambda}} = -1.8\log_{10}\left[\frac{6.9}{\text{Re}} + \left(\frac{k}{3.7\cdot D}\right)^{1.11}\right]. \qquad [7]$$

The total pressure drop in steady two-phase flow, when the accelerational effects are assumed very small and, hence, neglected, is

$$\left(\frac{\mathrm{d}p}{\mathrm{d}z}\right)_{\text{total}} = \left(\frac{\mathrm{d}p_f}{\mathrm{d}z}\right)_L + (1-\epsilon)\rho_L\cdot g = \left(\frac{\mathrm{d}p_f}{\mathrm{d}z}\right)_G + \epsilon\cdot\rho_G\cdot g. \qquad [8]$$

Equation [7] gives the frictional pressure drop ($\mathrm{d}p_f/\mathrm{d}z$) for both phases, and this equation is implicit. The best way to solve this equation, is to solve it for the film thickness.

The wall shear stress is:

$$\tau_0 = \frac{d}{4}\left(\frac{\mathrm{d}p_f}{\mathrm{d}z}\right)_L$$

and the interfacial shear stress is:

$$\tau_i = \left(\frac{\mathrm{d}-2\delta}{4}\right)\left(\frac{\mathrm{d}p_f}{\mathrm{d}z}\right)_G$$

The average pressure drop for a pipe with length, L, is:

$$\frac{\overline{\Delta p}}{\Delta z} = \frac{1}{L}\int_0^L \frac{\mathrm{d}p}{\mathrm{d}z}\,dz.$$

Figures 1 and 2 shows the comparison of air–water data from Cousins & Hewitt (1968) with calculation from the present model and the agreement is found to be good. Figures 3 and 4 shows the comparison of air–water data from Ueda & Nose (1974) and figure 4 shows how the present model predict the film thickness and thereby the void fraction. The present model predicts these data well.

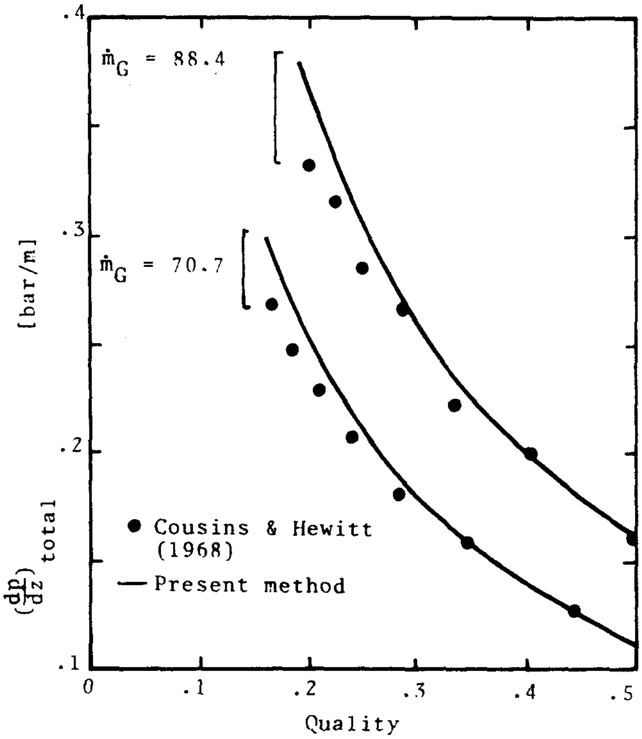

Figure 1. A comparison of pressure drop for air–water. Pressure 1.38 bar.

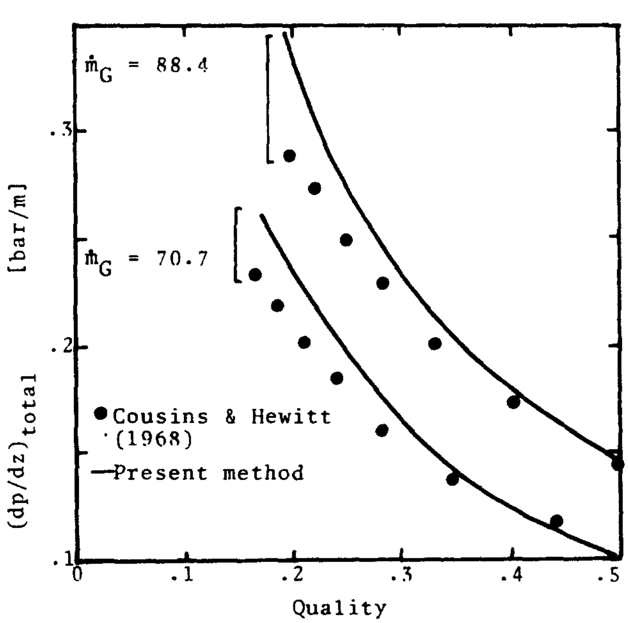

Figure 2. A comparison of pressure drop for air–water. Pressure 1.73 bar.

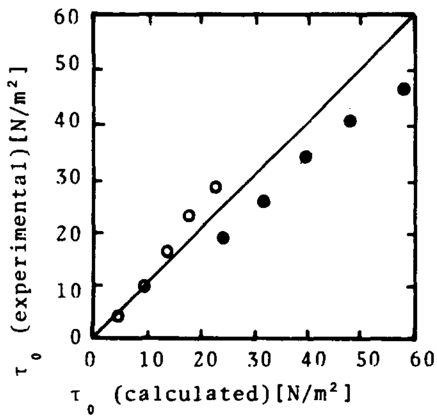

Figure 3. A comparison of wall shear stress for air-water data from Ueda & Nose (1974).

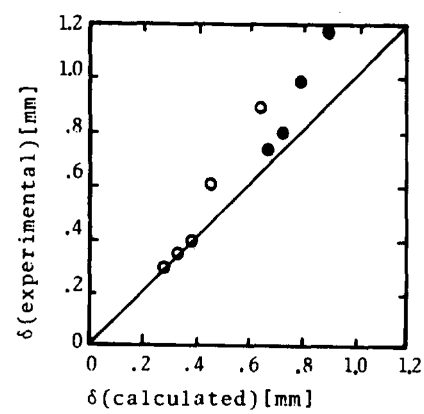

Figure 4. A comparison of the film thickness for air-water data from Ueda & Nose (1974).

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